Probability

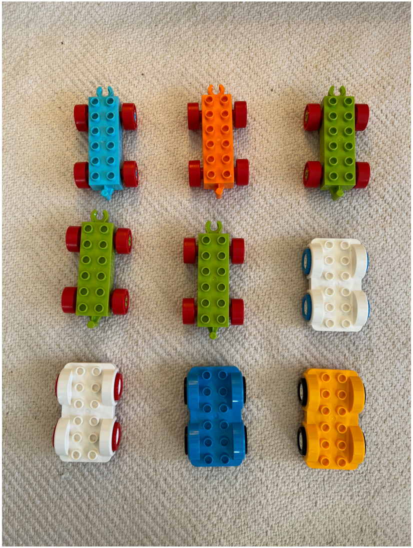
- Based on our sample or other random process (as in the coin flipping or a toddler choosing Lego), we would like to make valid statements about the underlying population or quantity of interest
- Probability is one tool that will help us do that
- Probability is all about talking about the chance of something (an event happening or observing a particular thing)
- There is uncertainty associated with the event or observation, and probability helps us to quantify this

Definitions

- **Experiment**: An experiment can be any process, in a laboratory or otherwise, where we can observe the result of a process and the result of that process is uncertain.
- **Events**: things that can happen
  - what's an example of an event when flipping a coin once? Four times?
  - what's an example of an event of sampling six people's heights?
- **Probability function**: a rule that assigns a value $P(A)$ to each event A. We know
  - Probability is positive
  - Probability is at most 1
  - The sum of probablities of all possible events is 1

Lego example

We have the following lego trains and cars:



$P(A \text{ or } B)$

chooses train or blue

$= \frac{6}{9}$

$= 2/3$

$P(B|A)$

$= \frac{1}{5}$

Lego example

My son randomly draws out one vehicle

P(A) = probability choose a train

$= \frac{5}{9}$

P(B) = probability choose blue

$= \frac{2}{9}$

$\frac{\#\text{ successes}}{\#\text{ possibilities}}$

Lego example

Let's define some events:

- A = "Choose a train"
- B = "Choose a vehicle that is blue"

What is $P(A)$? What is $P(B)$?

Probability is just counting!

Probability is just counting!

Additive / Union rule

What is $P(A \text{ or } B)$? That is the probability that the vehicle is a train or is the color blue?

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Note that if A and B are **mutually exclusive** then they can't happen together so $P(A \text{ or } B) = P(A) + P(B)$.

Conditional probability

"given"

Prob that the vehicle is blue given its a train

- The probability of something happening given we know something else
- $P(B|A)$ is conditional probability i.e. the probability of B given that A is true
- Lego examples
  - what is $P(B|A)$?
  - what is the probability that the vehicle is a train given it has red wheels?
  - what is the probability that the vehicle is white given it is a car?

Conditional probability

Conditional probability is important for us

- What's the probability that someone work's remotely given they work in finance (vs hospitality?)
- What's the probability that someone graduates college given their parent's did?

Multiplicative / Intersection rule

What is $P(A \text{ and } B)$? That is the probability that the vehicle is a train and is the color blue?

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

$$\frac{\cancel{5}}{9} \times \frac{1}{\cancel{5}} = \frac{1}{9}$$

Independence

If two events A and B are independent, then $P(A)$ is not affected by the condition B, and vice versa, so we can say that $P(A|B) = P(A)$ and likewise, $P(B|A) = P(B)$, so the multiplicative rule becomes

$$P(A \text{ and } B) = P(A) \times P(B)$$

Complements

the complement of any event A is the event [not A], i.e. the event that A does not occur. It is denoted A^c.

Lego practice

$$P(A) + P(A^c) = 1$$

$$P(A^c) = 1 - P(A)$$

Interpret and calculate the following

- $P(B|A)$
- $P(A|B)$
- $P(A^c)$ probability of a car $= \frac{4}{9}$
- $P(A|B^c)$