Sociology Quant Camp

Math review

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Hello

- Jointly appointed statistics and sociology
- Demographer
- Australian
- Teaching IDA in Winter
- Office in stats only: 9135

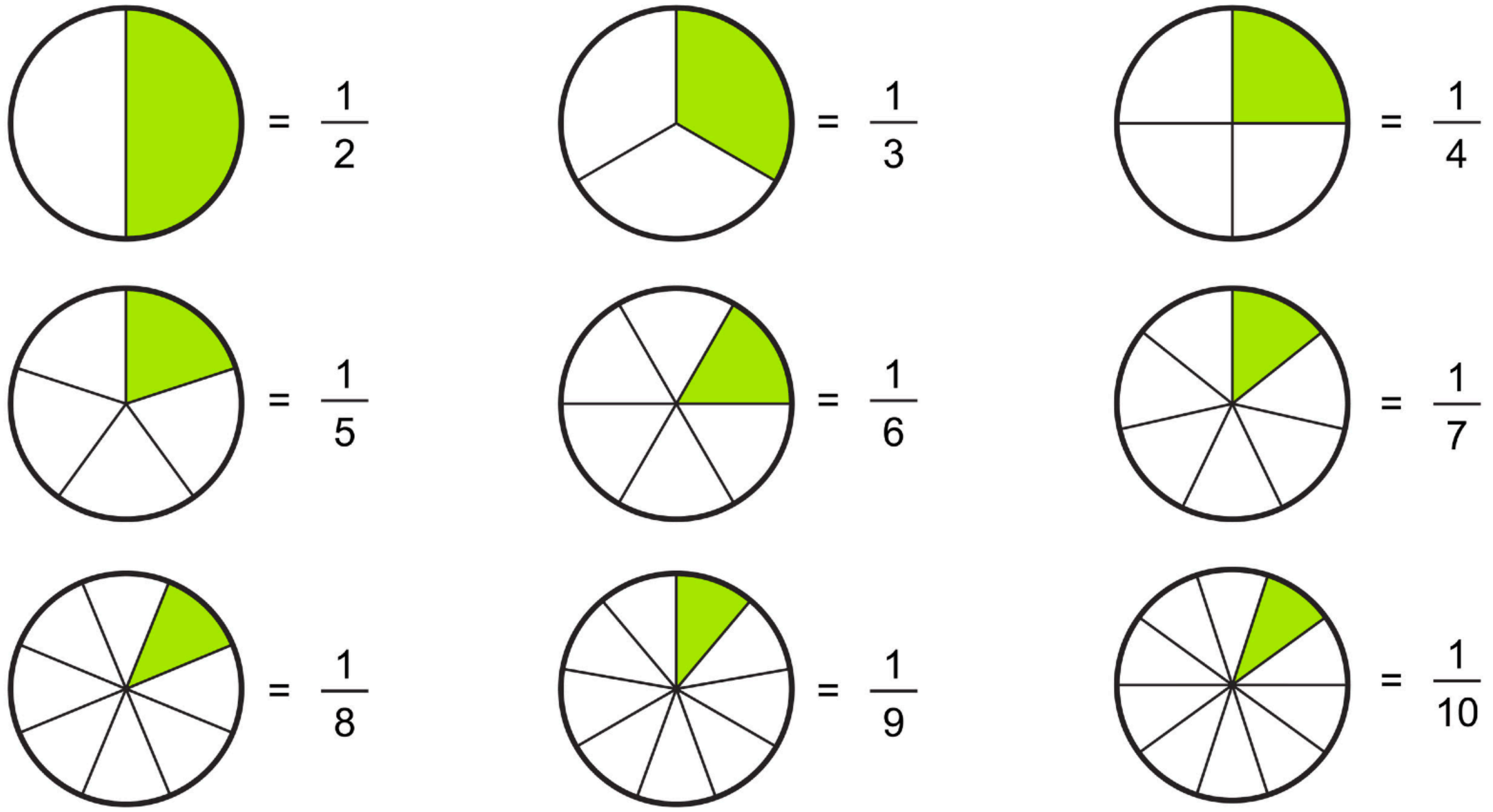
Overview

- Fractions, proportions, percents
- Mathematical notation
- Transformations
- Functions and graphing
- Algebraic manipulation

Fractions, proportions, percents

Fractions

- A fraction is a part of a whole
- We express a fraction as a quotient
- The top number is the **numerator**
- The bottom number is the **denominator**



$\frac{2}{4}$ $\frac{3}{9}$ $\frac{1}{3}$

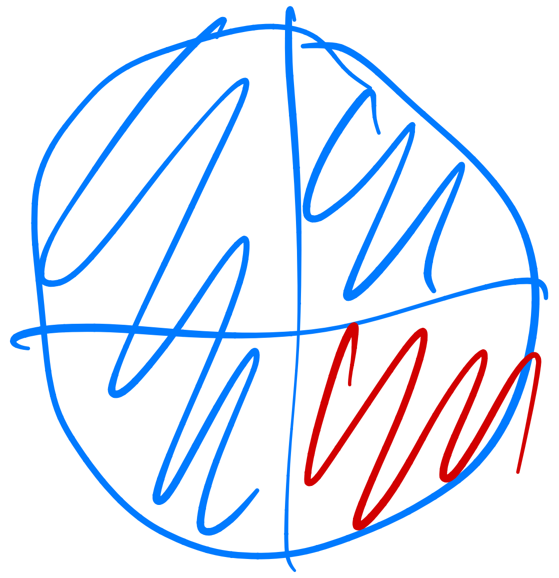
E.g. $\frac{6}{7}$

6 ← numerator

7 ← denominator

Questions

$5 \times \frac{3}{4} = \frac{15}{4} = 3\frac{3}{4}$



"half of 3/4"

- What is $\frac{2}{4}$ equivalent to? $\frac{1}{2}$
- What is $\frac{1}{2} \times \frac{3}{4}$? $= \frac{3}{8}$ $\frac{1}{2} \times \frac{2}{8} = \frac{2}{16} = \frac{1}{8}$
- What is $\frac{1}{4} + \frac{3}{4}$? $= \frac{4}{4} = 1$
- What is $\frac{1}{2} + \frac{3}{4}$? $\frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}$

Proportions

- A part, share, or number considered in relation to the whole
- Another way of writing fractions
- Rather than expressing as a quotient, calculate the quotient
- E.g. $\frac{1}{2} = 0.5$
- $\frac{2}{5} = 0.4$
- $\frac{1}{3} = 0.333...; \frac{2}{3} = 0.666...$

Proportions

- Note that computers represent fractions as proportions (e.g. try typing 1/4 into R)
- You probably won't have to ever calculate these by hand (but maybe you will)
- Trick to calculating by hand: get the denominator to equal 10, then the proportion is the numerator with the decimal point moved to the left
  - E.g. 1/2 —> 1/2 * 5/5 —> 5/10 —> 0.5
  - 1/4 —> 1/4*2.5 —> 2.5/10 —> 0.25
  - 2/3 —> 2/3* $\frac{10}{3}/\frac{10}{3}$ —> $\frac{20}{3}/10$ —> 0.667

Percents

- A number expressed as a fraction of 100
  - E.g. 1/2 —> 50/100 —> 50%
  - 4/5 —> 80/100 —> 80%
- Questions
  - What is 7/8 as a percent? $\frac{1}{8} = 0.125$ ✓ 87.5%
  - What is the proportion 0.64 as a percent? 64%

Percents

10% of 70 is 7

$3 \times 7 = 21$

- Questions:
  - What is 50% of 70? 35
  - What is 30% of 70? 21
  - 40% of a number is 7. What is the number?

$$\frac{7}{0.4}$$

Percent changes

$$\frac{150-100}{100} = \frac{50}{100} \qquad \frac{end - start}{start} \times 100$$

- In year 1 there were 100 students. In year 2 there were 150 students. What's the percentage change in students?

- My child's class size when from 30 to 40 kids this year. What's the percentage change in kids?

$$\frac{40-30}{30} \times 100 = \frac{10}{30} \times 100 = \frac{1}{3} \times 100 = 33.33\%$$

- It used to take me 60mins to run 10km. Now it takes me 50mins. What's the percentage change in time?

$$\frac{50-60}{60} = \frac{-10}{60} = \frac{-1}{6} = -16.67\%$$

Mathematical notation

Mathematical notation

$2 + 2 = 4$

- Lots of symbols representing operations, numbers, relations, and other objects and formulas
- Like a language, summarizing mathematical operations
- E.g. equals =, multiply x, add +, take away -
- But there are some symbols that are important that you may not be so familiar with

Letters as symbols

- Often in mathematics and statistics, we use certain letters to represent numbers in a general sense
- E.g. pretend you are interested in counting the number of cars that pass the university/college intersection over a half an hour period
  - You could collect **data (observations)** based on standing at the intersection and counting the number of cars in successive half hour periods
  - E.g. over three hours you may see 12, 36, 50, 20, 40, 50 cars
  - But **even before** observing the number of cars, we can talk about this concept (variable) in a general sense

Letters as symbols

- Define our variable of interest to be the number of cars that pass the university/college intersection in half an hour
- We can give this variable the symbol x
- x represents the number of cars observed in a half an hour period
- If we collect multiple observations of x, then we can add a subscript
- $x_1, x_2, x_3, \ldots$

Letters a symbols

- The letters x, y, z are the most common symbols used to denote numbers in a general way
- Blame Descartes (1637)

More symbols

- Imagine we have a set of 6 car observations $x_1, x_2, x_3, x_4, x_5, x_6$
- Now imagine we want to calculate the sum of those 6 observations
- We could write the sum as

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

- But there’s an easier way!

Summation

- $x_1 + x_2 + x_3 + x_4 + x_5 + x_6$ is the same as

$$\sum_{i=1}^{6} x_i$$

Sum the values of x_i from i equals 1 to $i = 6$

to

capital sigma

"sum"

from

general index

little sigma

σ ← standard deviation

μ "mu"

mean

- another thing you'll see a lot is the letter n

- n: used to denote sample size

- car example: n = 6

$$\frac{\sum_{i=1}^{n} x_i}{n}$$ (sum of everything) = mean/average = $\bar{x}$

i respondent #	x income	age	educ	hours worked
1				
2				
3				
4				
5				

Transformations

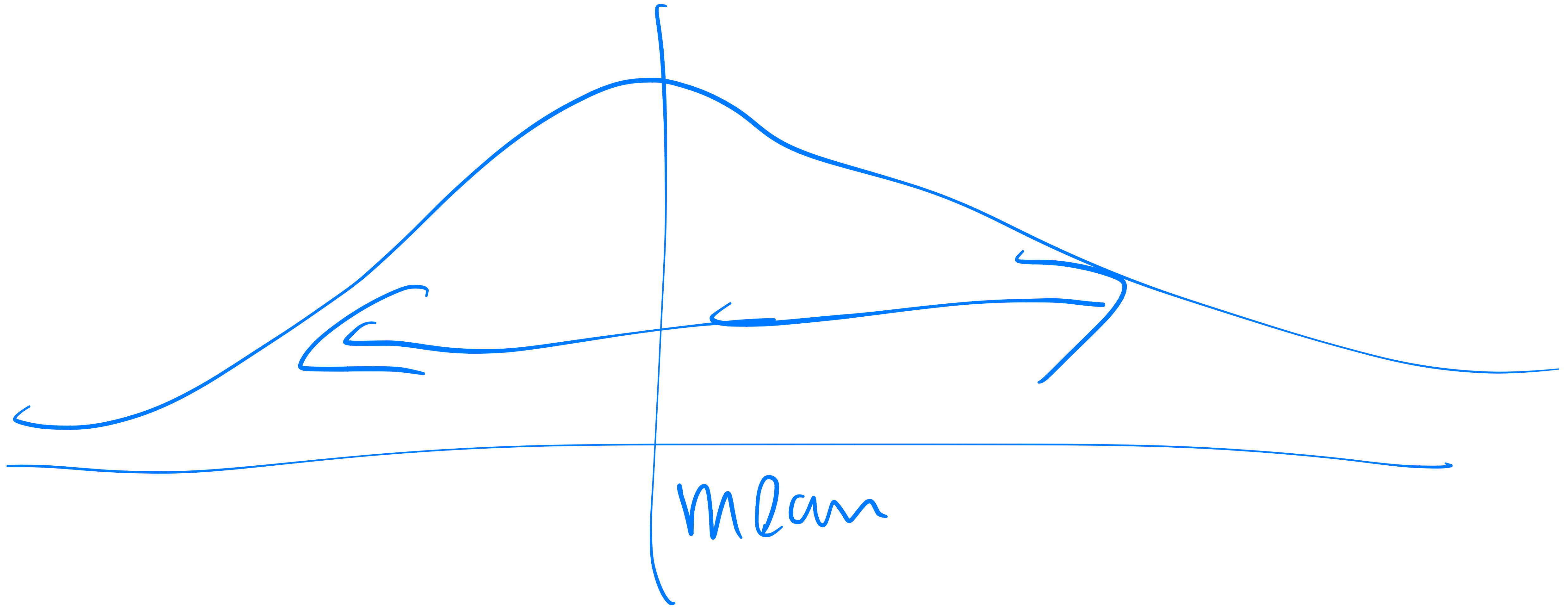
Transformations

- Transformations take a number and do something to it to get another number
- There are many different transformations, but let's cover to of the most important ones (in the context of regression)

1. Squaring/square roots

2. Exponentials / Logarithms

Squaring



- Multiply the number by itself
- Represented in **mathematical notation** as x^2
- E.g. $6^2 = 6 \times 6 = 36$
- You can calculate in R using x^2
- Questions:
  - What is 2^2? 4
  - What is $(-2)^2$? 4
  - Why is squaring an important transformation?

```
> 89^2
[1] 7921
```

(-89)^2

Square roots

- The inverse, or opposite operation of squaring a number is to find a number's **square root**
- Represented in mathematical notation as $\sqrt{x}$
- e.g. the square root of 9 is 3: $\sqrt{9} = 3$
- Notice that the square root of a number squared is just the number i.e. $\sqrt{x}^2 = x$
- You can calculate in R using the function sqrt(x)

```
> sqrt(8)
[1] 2.828427
```

Exponentials

e^{-2} positive number

- The number e is a mathematical constant, equal to about 2.71828
- (Think pi, $\pi = 3.1415...$)
- Another important transform is e^x
- E.g. $e^1 = e$ $e^0 = 1$
- $e^2 = 7.389$
- Calculate in R using the exp() function

```
> exp(2)
[1] 7.389056
```

Logarithms

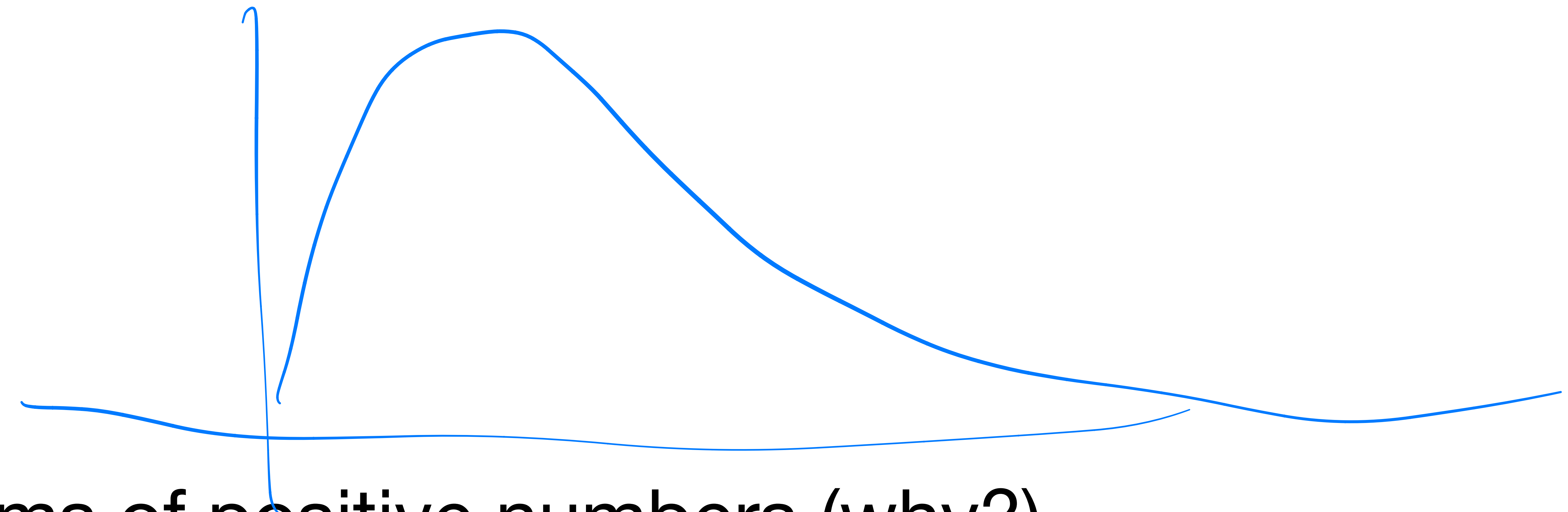
b^x $e^0 = 1$

- The inverse function of exponentiation is the logarithm
- That means that the logarithm of a number x to the base b is the exponent to which b must be raised to produce x.
- For example, $10^2 = 100$. so $\log 100$ to the base 10 is 2. Also written as $\log_{10} 100 = 2$.
- Importantly, natural logarithms are logs to the base e
- E.g. $\log e = 1; \log e^2 = 2$

$\log_e x = -$

$\log 1 = 0$

Logarithms



- Note! You can only take logarithms of positive numbers (why?)
- The transformation $\log x$ has the effect of "stretching out" smaller numbers and "shrinking" larger numbers
- Note! When x is less than 1, $\log x < 0$
- E.g. $\log 0.5 = -0.69$

$\log \frac{1}{2} = -0.69$

$\log 2 = +0.69$

```
> log(0.5)
[1] -0.6931472
```

Functions and graphing

Functions

- An expression, rule, or law that defines a relationship between one variable (the independent variable) and another variable (the dependent variable).
- For example
  - $y = x^2$
  - $y = \log x$
  - $y = mx + c$

Functions

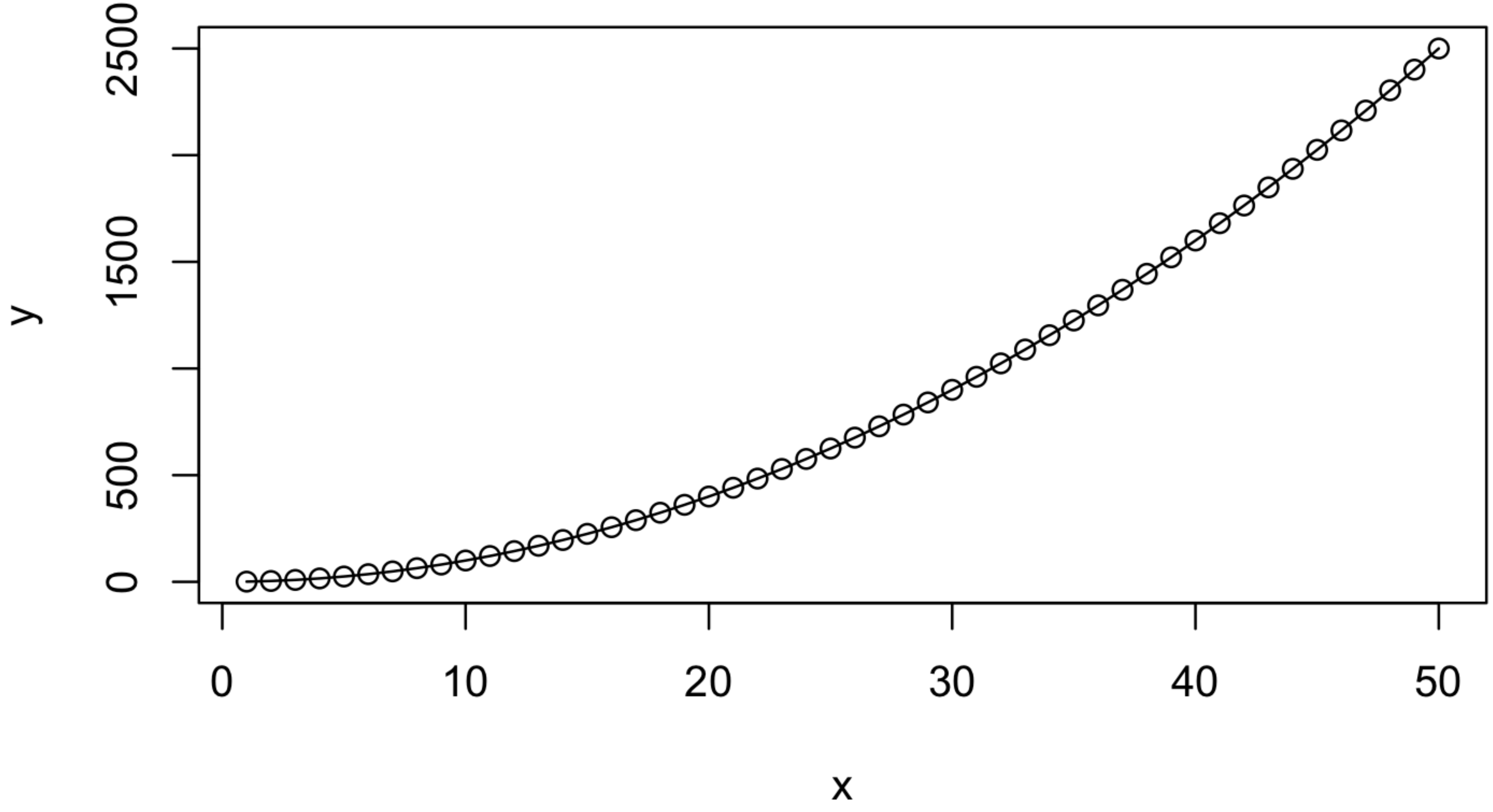
- Let's consider $y = x^2$
- We can map values of x to values of y

x	y
0	1
1	1
2	4
3	9

Graphing

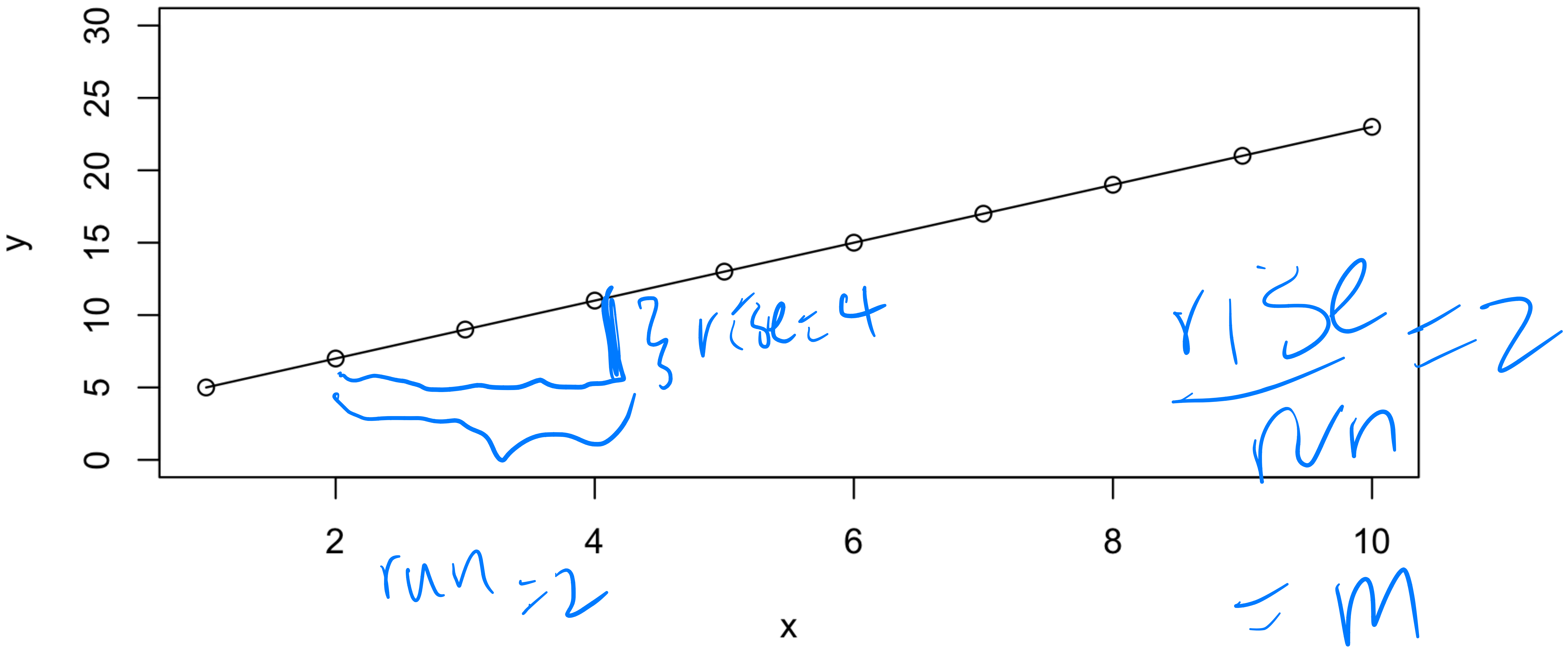
- We can then draw the function to see what it looks like

Graphing



Graphing

- $y = mx + c$ is the equation for a straight line
- c is the intercept
- m is the slope
- E.g m = 2, c = 3 —>



Graphing in R

Algebraic manipulation

Rearranging equations

- Sometimes we may have an equation in terms of one variable, but want to rearrange in so it is expressed in terms of another variable.
- E.g. consider $y = mx + c$. What if we wanted to know what m was?
- To isolate m, need to perform the same operations to both sides of the equation until m is on its own

Rearranging equations

$$y = mx + c$$

$$y - c = mx + c - c$$

$$\frac{y - c}{x} = \frac{mx}{x}$$

$$m = \frac{y - c}{x}$$

Questions

standard deviation   mean

- If $X = \sigma Z + \mu$, what does Z equal?
- If $y = \dfrac{2}{5}x^2 - 7$, what does x equal?

$$Z = \frac{X - \mu}{\sigma}$$

$$y + 7 = \frac{2}{5}x^2$$

$$(y+7)\frac{5}{2} = x^2$$

$$x = \sqrt{(y+7)\frac{5}{2}}$$