Measures of premature life lost at a fixed level of life expectancy

Monica Alexander, University of Toronto

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Abstract

Comparing patterns of mortality across populations that have similar levels of life expectancy can give important insights into differing levels of premature mortality over time and across space. We propose two summary measures to compare the premature life lost at a fixed life expectancy level: the number of person-years lost before reaching the life expectancy at birth age, T_{e_0}, and temporary life disparity, $e^\dagger_{e_0}$. We then illustrate these measures using data related to US states.

1 Introduction

Life expectancy at birth, e_0, is one of the the most common summary measure of longevity used to compare mortality conditions across different populations. It is studied by demographers, UN agencies, and journalists alike and has recently gained attention as the effects of Covid-19 saw widespread declines in life expectancy (Aburto et al. 2022).

While it is generally accepted that, given two populations, the one with the higher life expectancy has better overall mortality conditions, it is less clear how to compare two populations that have the same level of life expectancy. Many survival curves can have the same area and thus the same e_0, but different shapes imply different levels of premature mortality. For example, in Figure 1, survival curves for two populations A and B with the same life expectancy are shown. The curve shapes are slightly different, with population A experiencing lower survival rates at earlier ages, and thus has higher premature mortality. Comparing survival curves at a fixed level of life expectancy can give important insights into comparing different conditions in populations over time, and also in a similar period in response to a major mortality event, such as the Covid-19 pandemic.

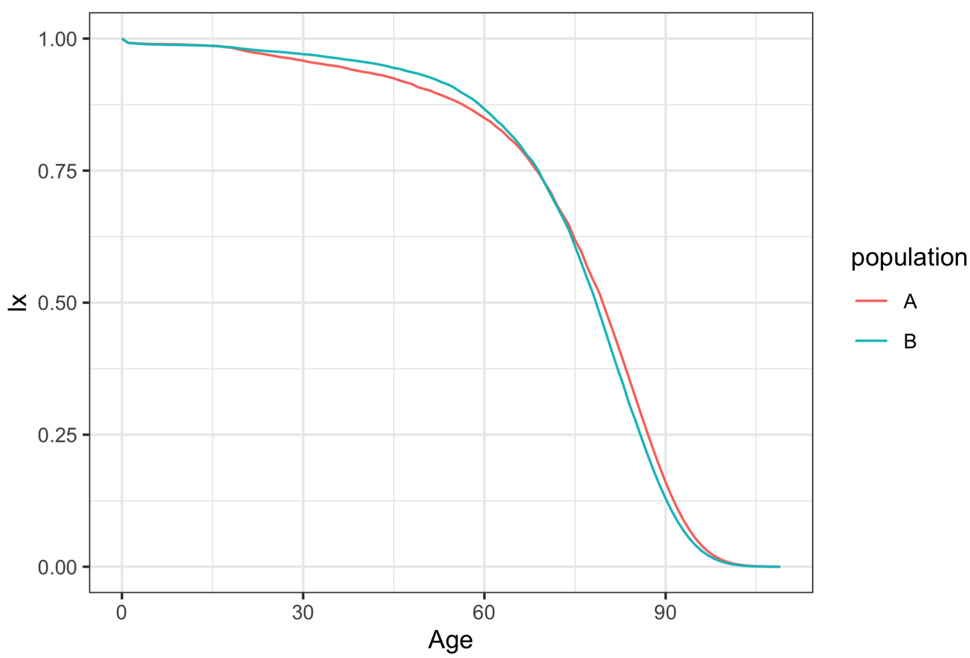

Figure 1: Two survival curves with equal life expectancy

In this abstract we introduce two summary measures of premature life lost at fixed life expectancy. We show that the person-years lived above age x when $x = e_0$, T_{e_0}, can be re-interpreted as the number of person-years lost before reaching the life expectancy at birth age. Additionally, we use a measure of 'temporary life disparity', $e^\dagger_{e_0}$, which can be interpreted as the years of life lost to death before reaching the life expectancy at birth age. We then illustrate these measures using data related to US states.

2 Measures of premature life lost at fixed life expectancy

2.1 Person-years lost before reaching e_0, T_{e_0}

Consider the survival curve shown in Figure 2. It has area under the curve, or life expectancy at birth, equal to e_0. The rectangle shown with the black line has the same area as the survival curve, also equal to e_0.

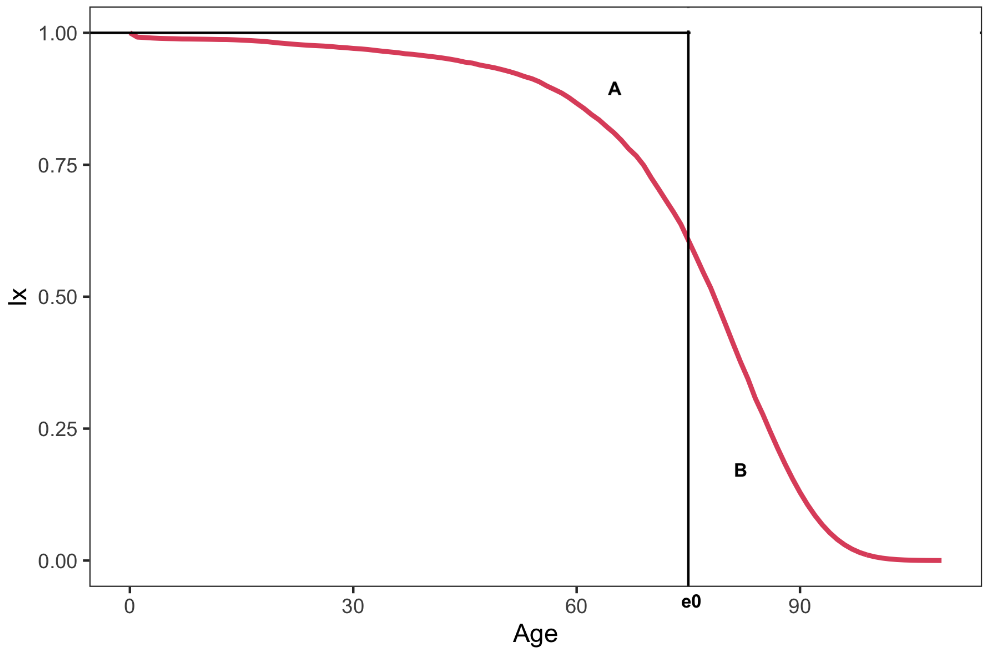

Figure 2: Survival curve compared to rectangle with same area

Our goal is derive a measure that summarizes differences in premature mortality across survival curves at fixed e_0. One such measure is the area A shown in Figure 2, which is equal to the person-years lost before reaching the population level e_0. From Figure 2, we know that

$$A = e_0 - \int_0^{e_0} l_x dx$$

We also know that

$$B = \int_{e_0}^{\omega} l_x dx = T_{e_0}$$

where ω is the oldest age. But as the area under the survival curve equals the area under the rectangle, which equals e_0, we also know that

$$A = B = T_{e_0}$$

Thus, the person-years lived above age x when $x = e_0$, T_{e_0}, can be re-interpreted as the number of person-years lost before reaching the life expectancy at birth age. This is a useful potential measure to compare populations at fixed life expectancies, with populations with higher values of T_{e_0} suggesting more premature mortality levels (and as such potentially worse underlying mortality conditions).

2.2 Temporary life disparity at e_0, $e^\dagger_{e_0}$

We define life disparity using $e^\dagger$, which is the average number of life-years lost as a result of death (Vaupel and Romo 2003):

$$e^\dagger = \int_0^\omega d_x e_x dx$$

Now define the temporary life disparity at life expectancy age e_0 to be

$$e^\dagger_{e_0} = \int_0^{e_0} d_x \frac{\int_x^{e_0} l_a da}{l_x} dx$$

This measure is defined in Andersen et al (2013) in the context of cause-specific life lost, although we focus on a fixed age e_0 here. It is the average number of life-years lost as a result of death before the overall life expectancy age, e_0. It is the remaining life expectancy left between the age when death occurs and the age of e_0. Thus, is it is an alternative measure for capturing the level of premature mortality, i.e., death before the expected age.

3 Illustration

We illustrate these two measures using data from the Human Mortality Database's US States database (https://usa.mortality.org/). We downloaded estimated life tables for males for the period 1959-2019. For illustration, we set the fixed life expectancy at birth of interest to be 75 years and then retained the survival curves for each state in the year where e_0 was around 75 years. The result is a data set of 43 states (Table 1). Hawaii reached a male life expectancy around 75 in 1985, whereas Missouri achieved this level almost 30 years later in 2014.

For each of these survival curves, we also calculated T_{e_0} and $e^\dagger_{e_0}$, also shown in Table 1. It can be seen that in general, the earlier the state reached the target life expectancy, the small both measures, suggesting that the 'leading' states had less premature mortality compared to those states reaching the target life expectancy more recently. The strong correlation with time can also be seen in the plots below (Figure 3). While lower-performing states have 'caught up' to previous life expectancy levels, mortality conditions for those in age groups below 75 are worse than they were when the target of 75 was first reached.

Table 1: Male life expectancies around 75, the year they were obtained, person-years lost before 75, and temporary life disparity at age 75.

State	Year	Life expectancy (years)	T(e0)	Temporary life disparity
HI	1985	75.0	6.20	5.81
UT	1989	75.0	5.96	5.66
MN	1995	75.0	5.93	5.60
WA	1996	74.7	5.90	5.83
ND	1996	75.1	5.93	5.47
CO	1996	75.0	6.06	5.70
NH	1996	75.0	5.60	5.30
MA	1997	75.3	5.87	5.28
OR	1997	74.8	5.92	5.78
ID	1997	74.9	6.04	5.84
IA	1997	75.1	5.90	5.48
CT	1997	75.1	6.05	5.64
CA	1998	75.0	6.09	5.68
VT	1998	75.0	5.87	5.53
WI	1998	75.0	5.89	5.56
NE	1999	75.0	5.96	5.65
KS	2000	74.9	5.94	5.75
SD	2000	75.0	6.16	5.85
WY	2000	75.0	6.01	5.68
NJ	2001	74.7	5.93	5.87
NY	2001	74.9	6.11	5.82
MT	2002	74.9	6.29	5.99
ME	2002	75.0	5.82	5.52
RI	2002	75.0	6.04	5.71
VA	2004	75.3	6.22	5.62
AK	2004	75.0	6.66	6.33
IL	2004	75.0	6.20	5.85
PA	2006	74.8	6.34	6.14
TX	2006	75.2	6.41	5.88
FL	2006	75.1	6.85	6.35
MI	2006	75.1	6.41	5.96
AZ	2006	75.1	6.78	6.31
DE	2006	75.0	6.54	6.17
MD	2006	75.0	6.55	6.13
NV	2008	74.8	6.20	6.01
NC	2009	74.9	6.39	6.07
OH	2009	75.0	6.41	6.00
IN	2010	75.0	6.37	5.99
DC	2011	75.0	6.78	6.32
GA	2013	75.1	6.44	5.96
NM	2014	75.0	7.15	6.68
MO	2014	75.0	6.57	6.13

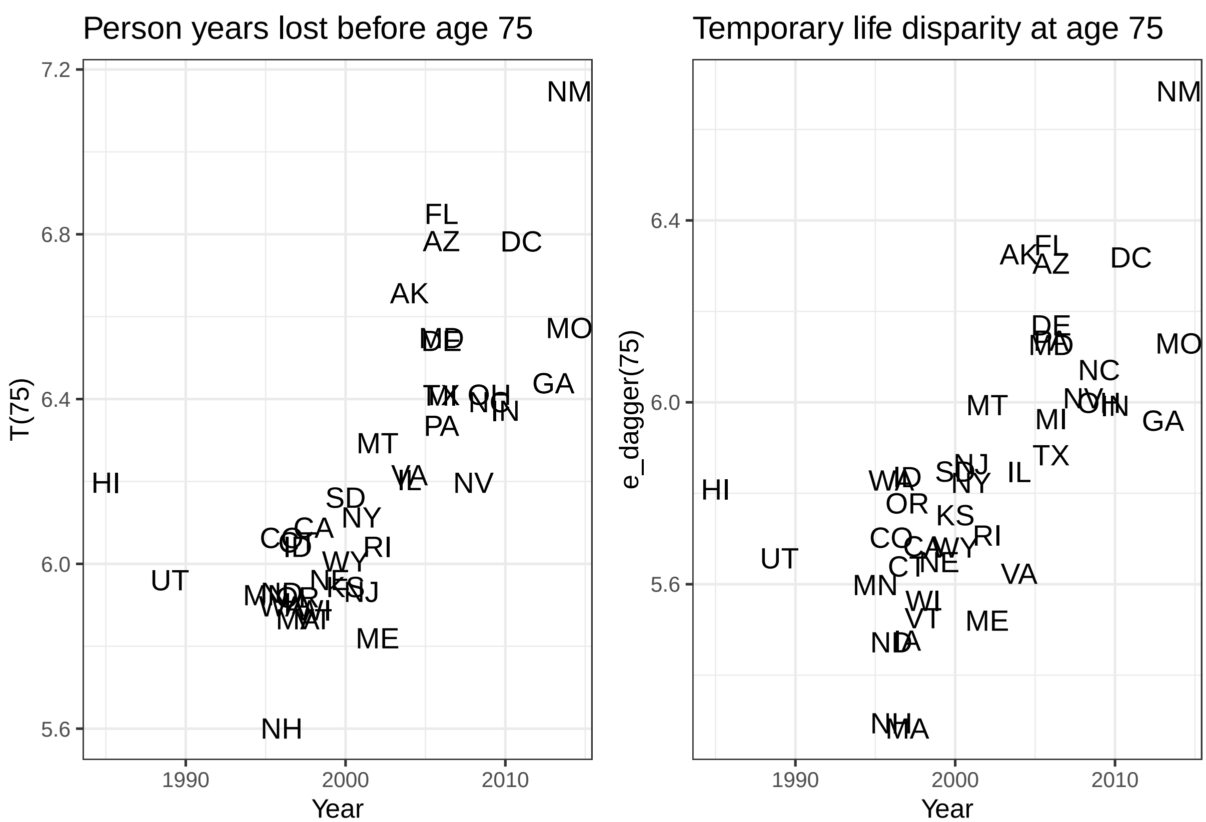

Figure 3: Plots of person years lost (left) and temporary life disparity (right) versus year reached life expectancy of 75

4 Future work

Future work will compare T_{e_0} and $e^{\dagger}_{e_0}$ to other summary measures to better understand their relationship. In addition, we will also focus on the application of these measures to survival curves across countries and other areas since the onset of the Covid-19 pandemic, to better understand differences across geographies.

References

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